Optical solitons, similaritons and rogue waves

We have been interested in the formation of optical structures, such as solitons and similaritons, in open nonlinear systems. While solitons are static structures resulting from the balance between the medium nonlinearity and linear dispersion or diffraction on the one hand, and gain and loss in nonlinear media on the other, similaritons, which is a jargon for stable self-similar waves, are dynamical entities maintaining their structure in said media and simply scaling on propagation there. We have discovered a rich class of soliton-like similaritons in open nonlinear systems with Kerr nonlinearities, uncompensated linear gain and linear inhomogeneity [1, 2]. The latter can be realized by linear refractive index grading in optical waveguides or trapping potential in matter-wave systems such as Bose-Einstein condensates.

The original, and rightfully dreaded, ocean rogue waves manifest themselves as the waves of unusually high amplitudes, and therefore destructive power, on the surface of deep water; rogue waves have a statistical nature because they occur randomly, yet more often than is anticipated by generic Gaussian statistics. Most work on the extreme event appearance in nonlinear wave systems has focused on rogue wave excitation through modulation instability (MI) of a continuous wave background. In most realistic situations, however, there exists an external driving source such as random impurities in optical and/or matter wave systems, or random bursts of wind in the case of ocean rogue waves. We have recently shown [3] that under favourable conditions, the external driving acts in concert with MI to facilitate extreme event excitation in nonlinear wave systems. This can become particularly pronounced if the external factors cause random focusing of the waves, thereby further promoting high-amplitude wave events.

---

[1] S. A. Ponomarenko and G. P. Agrawal, "Do Solitonlike Self-Similar Waves Exist in Nonlinear Optical Media?" *Phys. Rev. Lett.*, **97**, 013901 (2006).

[2] S. A. Ponomarenko and G. P. Agrawal, "Optical similaritons in nonlinear waveguide amplifiers", *Opt. Lett.*, **32**, 1659 (2007).

[3] S. Alhadlaq and S. A. Ponomarenko, "Emergence of extreme events from randomly chirped condensate," *Phys. Lett. A*, **526**, 129958 (2024).